

Solutions to Practice Problems for Test #5

I. Invalidity

1. There is a counterexample in a 1-member universe, where:
Aa: True; Ba: False; Ca: False
2. There is a counterexample in a 2-member universe, where:
Ea: True; Fa: False; Ga: False
Eb: True or False; Fb: True; Gb: True
3. There is a counterexample in a 2-member universe, where:
Pa: True; Qa: False; Ra: False
Pb: False; Qb: True; Rb: False
4. There is a counterexample in a two-member universe where:
Pa: True; Qa: False; Ra: False
Pb: True; Qb: True; Rb: False
5. There is a counterexample in a three-member universe where:
Aa: True; Ba: False; Ca: True
Ab: False; Bb: True; Cb: True
Ac: False; Bc: False; Cc: False

Alternative solutions are possible.

II. Translation in **F**.

1. $(\forall x)(Ax \supset Px)$
2. $(\forall x)[(Tx \cdot Ax) \supset Px]$
3. $(\forall x)[(Tx \cdot Ax) \supset Wxn]$
4. $(\forall x)[(Tx \cdot Ax) \supset (\exists y)(Py \cdot Wxy)]$
5. $(\forall x)\{(Tx \cdot Ax) \supset (\exists y)[(Py \cdot By) \cdot Wxy]\}$
6. $(\forall x)\{(Bx \cdot Ax) \supset \sim(\exists y)[(Ty \cdot Py) \cdot Rxy]\}$
or: $(\forall x)\{(Bx \cdot Ax) \supset (\forall y)[(Ty \cdot Py) \supset \sim Rxy]\}$
7. $(\exists x)\{(Tx \cdot Px) \cdot (\forall y)[(By \cdot Py) \supset \sim Rxy]\}$
8. $(\forall x)(Ax \supset Mnx)$
9. $(\forall x)(Ppx \supset Mnx)$
10. $(\forall x)(Wxn \supset Mnx)$
11. $(\forall x)(\sim Mxx \supset Mnx)$
12. $(\forall x)(\forall y)(Wxy \supset \sim Wyx)$
13. $(\forall x)\{(Ax \cdot Px) \supset (\exists y)[(Ty \cdot Py) \cdot Rxy]\} \supset (\exists x)[Tx \cdot (\forall y)(Ay \supset Wxy)]$

III. Derivations in F.

1. 1. $(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy$
 2. $(\exists x)(\forall y)\sim Bxy$ / $(\exists x)(\forall y)\sim Axy$
 3. $\sim(\forall x)(\exists y)Bxy$ 2, QE, QE
 4. $\sim(\forall x)(\exists y)Axy$ 1, 3, MT
 5. $(\exists x)(\forall y)\sim Axy$ 4, QE, QE

QED

2. 1. $(\forall x)(Ax \supset Bx)$ / $(\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$
 2. $(\exists y)(Ay \cdot Cxy)$ ACP
 3. $Aa \cdot Cxa$ 2, EI
 4. Aa 3, Simp
 5. $Aa \supset Ba$ 1, UI
 6. Ba 5, 4, MP
 7. Cxa 3, Com, Simp
 8. $Ba \cdot Cxa$ 6, 7, Conj
 9. $(\exists z)(Bz \cdot Cxz)$ 8, EG
 10. $(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)$ 2-9, CP
 11. $(\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ 10, UG

QED

3. 1. $\sim(\exists x)(Axa \cdot \sim Bxb)$
 2. $\sim(\exists x)(Dxd \cdot Dbx)$
 3. $(\forall x)(Bex \supset Dxd)$ / $\sim(Aea \cdot Dgd)$
 4. $Aea \cdot Dgd$ AIP
 5. $(\forall x)\sim(Axa \cdot \sim Bxb)$ 1, QE
 6. $(\forall x)(\sim Axa \vee Bxb)$ 5, DM, DN
 7. $\sim Aea \vee Beb$ 6, UI
 8. Aea 4, Simp
 9. Beb 7, 8, DN, DS
 10. $(\forall x)\sim(Dxd \cdot Dbx)$ 2, QE
 11. $(\forall x)(\sim Dxd \vee \sim Dbx)$ 10, DM
 12. $\sim Dgd \vee \sim Dbd$ 11, UI
 13. Dgd 4, Com, Simp
 14. $\sim Dbd$ 12, 13, DN, DS
 15. $Beb \supset Dbd$ 3, UI
 16. Dbd 15, 9, MP
 17. $Dbd \cdot \sim Dbd$ 16, 14, Conj
 18. $\sim(Aea \cdot Dgd)$ 4-17, IP

QED

- 4.
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|---|-------------------------------------|
| 1. $(\forall x)\{(Px \cdot Qx) \supset (\exists y)[(Py \cdot Qy) \cdot \sim Rxy]\}$ | |
| 2. $(\forall x)[Px \equiv (Qx \cdot Tx)]$ | |
| 3. $(\forall x)[Px \supset (\forall y)(Sy \supset Ryx)]$ | $/ (\forall x)(Px \supset \sim Sx)$ |
| 4. Px | ACP |
| 5. $Px \equiv (Qx \cdot Tx)$ | 2, UI |
| 6. $Qx \cdot Tx$ | 5, 4, BMP |
| 7. Qx | 6, Simp |
| 8. $Px \cdot Qx$ | 4, 7, Conj |
| 9. $(Px \cdot Qx) \supset (\exists y)[(Py \cdot Qy) \cdot \sim Rxy]$ | 1, UI |
| 10. $(\exists y)[(Py \cdot Qy) \cdot \sim Rxy]$ | 9, 8, MP |
| 11. $(Pa \cdot Qa) \cdot \sim Rxa$ | 10, EI |
| 12. $Pa \supset (\forall y)(Sy \supset Rya)$ | 3, UI |
| 13. $Pa \cdot Qa$ | 11, Simp |
| 14. Pa | 13, Simp |
| 15. $(\forall y)(Sy \supset Rya)$ | 12, 14, MP |
| 16. $Sx \supset Rxa$ | 15, UI |
| 17. $\sim Rxa \cdot (Pa \cdot Qa)$ | 11, Com |
| 18. $\sim Rxa$ | 17, Simp |
| 19. $\sim Sx$ | 16, 18, MT |
| 20. $Px \supset \sim Sx$ | 4-19, CP |
| 21. $(\forall x)(Px \supset \sim Sx)$ | 20, UG |

QED

- 5.
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|---|--------------------|
| 1. $(\forall x)(\forall y)(Axy \equiv Ayx)$ | |
| 2. $(\forall x)(\forall y)(\forall z)[(Axy \cdot Ayz) \supset Axz]$ | |
| 3. $(\exists x)(\exists y)Axy$ | $/ (\exists x)Axx$ |
| 4. $(\exists y)Aay$ | 3, EI |
| 5. Aab | 4, EI |
| 6. $(\forall y)(Aay \equiv Aya)$ | 1, UI |
| 7. $Aab \equiv Aab$ | 6, UI |
| 8. Aba | 7, 5, BMP |
| 9. $(\forall y)(\forall z)[(Aay \cdot Ayz) \supset Aaz]$ | 2, UI |
| 10. $(\forall z)[(Aab \cdot Abz) \supset Aaz]$ | 9, UI |
| 11. $(Aab \cdot Aba) \supset Aaa$ | 10, UI |
| 12. $Aab \cdot Aba$ | 5, 8, Conj |
| 13. Aaa | 11, 12, MP |
| 14. $(\exists x)Axx$ | 13, EG |

QED

- 6.
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|--|----------|
| 1. $(\forall x)[(\exists y)Fxy \supset (\forall z)(Gz \supset Fxz)]$ | |
| 2. Fab | |
| 3. Gc | / Fac |
| 4. $(\exists y)Fay$ | 2, EG |
| 5. $(\exists y)Fay \supset (\forall z)(Gz \supset Faz)$ | 1, UI |
| 6. $(\forall z)(Gz \supset Faz)$ | 5, 4, MP |
| 7. $Gc \supset Fac$ | 6, UI |
| 8. Fac | 7, 3, MP |

QED

- 7.
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|---|---|
| 1. $(\forall x)(\forall y)[Nxy \equiv (Px \cdot Py)]$ | |
| 2. $(\forall x)(Ox \supset Px)$ | |
| 3. $(\exists x)(\exists y)[(Ox \cdot Oy) \cdot Myx]$ | / $(\exists x)(\exists y)(Myx \cdot Nxy)$ |
| 4. $(\exists y)[(Oa \cdot Oy) \cdot Mya]$ | 3, EI |
| 5. $(Oa \cdot Ob) \cdot Mba$ | 4, EI |
| 6. $Oa \cdot Ob$ | 5, Simp |
| 7. Oa | 6, Simp |
| 8. $Oa \supset Pa$ | 2, UI |
| 9. Pa | 8, 7, MP |
| 10. $Ob \cdot Oa$ | 6, Com |
| 11. Ob | 10, Simp |
| 12. $Ob \supset Pb$ | 2, UI |
| 13. Pb | 12, 11, MP |
| 14. $Pa \cdot Pb$ | 9, 13, Conj |
| 15. $(\forall y)[Nay \equiv (Pa \cdot Py)]$ | 1, UI |
| 16. $Nab \equiv (Pa \cdot Pb)$ | 15, UI |
| 17. $(Pa \cdot Pb) \equiv Nab$ | 16, Com |
| 18. Nab | 17, 14, BMP |
| 19. $Mba \cdot (Oa \cdot Ob)$ | 5, Com |
| 20. Mba | 19, Simp |
| 21. $Mba \cdot Nab$ | 20, 18, Conj |
| 22. $(\exists y)(Mya \cdot Nay)$ | 21, EG |
| 23. $(\exists x)(\exists y)(Myx \cdot Nxy)$ | 22, EG |

QED

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|----|---|-----------------------------------|
| 8. | 1. $(\forall x)\{Px \supset (\exists y)[Qy \cdot (Rxy \cdot Ryx)]\}$ | |
| | 2. $(\exists x)\{Px \cdot (\forall y)[Sy \supset (\sim Rxy \vee \sim Ryx)]\}$ | / $(\exists x)(Qx \cdot \sim Sx)$ |
| | 3. $Pa \cdot (\forall y)[Sy \supset (\sim Ray \vee \sim Rya)]$ | 2, EI |
| | 4. Pa | 3, Simp |
| | 5. $Pa \supset (\exists y)[Qy \cdot (Ray \cdot Rya)]$ | 1, UI |
| | 6. $(\exists y)[Qy \cdot (Ray \cdot Rya)]$ | 5, 4, MP |
| | 7. $Qb \cdot (Rab \cdot Rba)$ | 6, EI |
| | 8. $(\forall y)[Sy \supset (\sim Ray \vee \sim Rya)] \cdot Pa$ | 3, Com |
| | 9. $(\forall y)[Sy \supset (\sim Ray \vee \sim Rya)]$ | 8, Simp |
| | 10. $Sb \supset (\sim Rab \vee \sim Rba)$ | 9, UI |
| | 11. $Sb \supset \sim(Rab \cdot Rba)$ | 10, DM |
| | 12. $(Rab \cdot Rba) \cdot Qb$ | 7, Com |
| | 13. $Rab \cdot Rba$ | 12, Simp |
| | 14. $\sim\sim(Rab \cdot Rba)$ | 13, DN |
| | 15. $\sim Sb$ | 11, 14, MT |
| | 16. Qb | 7, Simp |
| | 17. $Qb \cdot \sim Sb$ | 16, 15, Conj |
| | 18. $(\exists x)(Qx \cdot \sim Sx)$ | 17, EG |

QED